Exercise 20

Find the limit or show that it does not exist.

$$\lim_{t\to\infty} \frac{t - t\sqrt{t}}{2t^{3/2} + 3t - 5}$$

Solution

Multiply the numerator and denominator by the reciprocal of the highest power of t in the denominator.

$$\begin{aligned}
\lim_{t\to\infty} \frac{t - t\sqrt{t}}{2t^{3/2} + 3t - 5} &= \lim_{t\to\infty} \frac{t - t\sqrt{t}}{2t^{3/2} + 3t - 5} \cdot \frac{\frac{1}{t^{3/2}}}{\frac{1}{t^{3/2}}} \\
&= \lim_{t\to\infty} \frac{(t - t\sqrt{t})\frac{1}{t^{3/2}}}{(2t^{3/2} + 3t - 5)\frac{1}{t^{3/2}}} \\
&= \lim_{t\to\infty} \frac{\frac{1}{t^{1/2}} - 1}{2 + \frac{3}{t^{1/2}} - \frac{5}{t^{3/2}}} \\
&= \frac{\lim_{t\to\infty} \left(\frac{1}{t^{1/2}} - 1\right)}{\lim_{t\to\infty} \left(2 + \frac{3}{t^{1/2}} - \frac{5}{t^{3/2}}\right)} \\
&= \frac{\lim_{t\to\infty} \frac{1}{t^{1/2}} - \lim_{t\to\infty} 1}{\lim_{t\to\infty} 2 + \lim_{t\to\infty} \frac{3}{t^{1/2}} - \lim_{t\to\infty} \frac{5}{t^{3/2}}} \\
&= \frac{0 - 1}{2 + 0 - 0} \\
&= -\frac{1}{2}
\end{aligned}$$